Math 53 – Worksheet 1
GSI: Oltman, (1/26/19)
10.4 (Area and Length in Polar Coordinates)

Formula:

$$Area = \frac{1}{2}\int_a^b r^2 d\theta \qquad\qquad Arc\ Length = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$$

Problem 1. *Sketch the curve $r = 3\cos\theta$ and find the area that it encloses.*

Solution 1. You can use the formula, but you may get the wrong answer (since this curve goes around twice from 0 to π. An easier solution is noting that this curve is the circle of radius 3/2 centered at 3/2, therefore has area $\pi 9/4$

Problem 2. *Find the area of the region that lies inside the first curve and outside the second curve. $r = 1 - \sin\theta, r = 1$.*

Solution 2. First find the intersection points which is:

$$\sin\theta = 0$$

therefore $\theta = 0, \pi$. Then you need to think about what region you must integrate, either from 0 to π or π to 2π. The correct answer is π to 2π (I suggest plotting the graphs, then this should make sense), then the are will be:

$$\frac{1}{2}\int_\pi^{2\pi} (1 - \sin\theta)^2 d\theta - \frac{1}{2}\int_\pi^{2\pi} d\theta = \frac{8 + \pi}{4}$$

Problem 3. *Compute the length of the polar curve: $r = 3\sin\theta$ for $0 \leq \theta \leq \pi/3$*

Solution 3.

$$\int_0^{\pi/3} \sqrt{9\sin\theta^2 + 9\cos\theta^2} d\theta = \int_0^{\pi/3} 3 d\theta = \pi$$

Problem 4. *Find the area of the region that lies inside the curve $r = \cos 2\theta$ but outside the curve $r = 2 + \sin\theta$*

Solution 4. This problem is not possible to do without a calculator.

Problem 5. *Compute the length of the curve: $r = 3\sin\theta$, $0 \leq \theta < \pi/3$*

Solution 5. This is a repeat problem

Problem 6. *Find the points on the curve where the tangent line is horizontal or vertical. $r = e^\theta$*

Solution 6. We would like to compute dy/dx with $y = r\sin\theta$ and $x = r\cos\theta$ and find the θ such that $dy/d\theta = 0$ (horizontal points) and when $dx/d\theta = 0$ (vertical points). Note that:

$$\frac{dy}{d\theta} = r'\sin\theta + r\cos\theta = e^\theta(\sin\theta + \cos\theta)$$

This is zero for $\theta = 3\pi/4$ and $7\pi/4$. And:

$$\frac{dx}{d\theta} = e^\theta(\cos\theta - \sin\theta)$$

which is zero for $\theta = \pi/4$ and $5\pi/4$